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CALCULATING ELECTRICAL REQUIREMENTS FOR DIRECT CURRENT ELECTRIC ACTUATORS

Joshua Stapp

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U.S. ARMY ARMAMENT RESEARCH, DEVELOPMENT AND ENGINEERING CENTER

Weapons and Software Engineering Center

Picatinny Arsenal, New Jersey

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14. ABSTRACT
When designing electro-mechanically actuated systems, there are several electrical design requirements that must be determined. The mechanical loads and power requirements translate to electrical requirements, taking into account multiple factors such as: efficiency losses, current draws, and regenerative power. These requirements lead to the determination of multiple design decisions such as: operating voltage, regenerative energy capture/dissipation, and conductor size, etc.

This report provides an overview of electrical factors that require consideration when designing an electrically actuated system. Overall theory is discussed, derivation of required calculations is described, and guidelines are provided to help make appropriate design decisions.

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SUMMARY

This report describes methods to calculate electrical requirements stemming from the use of electric actuators. Topics covered include: (1) calculating requirements for the power source, (2) determining current draws, and (3) the handling of regenerative energy.

INTRODUCTION

Servo control systems require accurate control of motion parameters to include: acceleration, velocity, and position. This requires a controller that can apply current (torque) to accelerate a motor in a given direction, as well as provide an opposing current to decelerate it. When this application of aiding and opposing torque can be carried out in both directions, it is referred to as four quadrant motor control as depicted in figure 1.

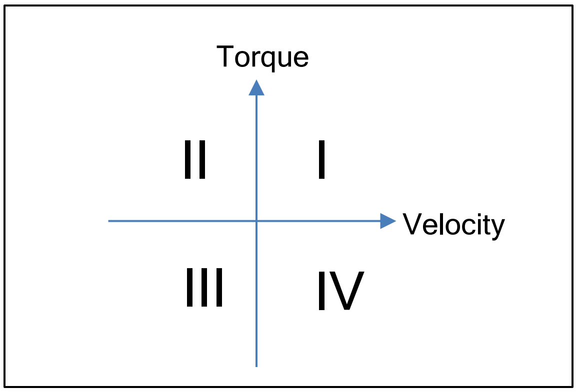

Figure 1
Four quadrant servo control

In four quadrant electric actuation systems, energy changes its form from electrical current flow to mechanical motion and vice versa. This conversion of energy is performed by an electric motor. An electric motor can be modeled electrically as a resistor, an inductor, and a voltage source. The resistor represents the resistance of the windings and internal wiring. The inductance is created from the turns of the wire that make up the windings. The voltage source is a result of the back electromotive force (EMF) created by the rotation of the motor shaft. When an electric motor shaft rotates, it produces an opposing voltage proportional to the motor's angular velocity. When the applied voltage exceeds the back EMF voltage, motoring occurs. When the back EMF voltage is greater than the applied voltage, braking occurs and the motor generates energy. In steady state, the difference between the applied voltage and the motor's back EMF, divided by the circuit's resistance, gives the current flowing in the motor windings. A motor's current is directly proportional to its mechanical output torque.

Figure 2 depicts the conversion of energy from electrical input to mechanical output. Electrical energy is input to a power supply. The power supply converts the input energy into a form that can be used by the motor drivers [i.e., alternating current (AC) to direct current (DC)]. The motor driver applies the energy from the supply to the motor as necessary to obtain the intended motion. The electric motor then converts the electrical energy into mechanical energy. The output of the motor is typically mated with some form of mechanical actuator that converts the motor's output to the intended motion. At each point in the conversion process, some energy is lost due to inefficiencies in the system.

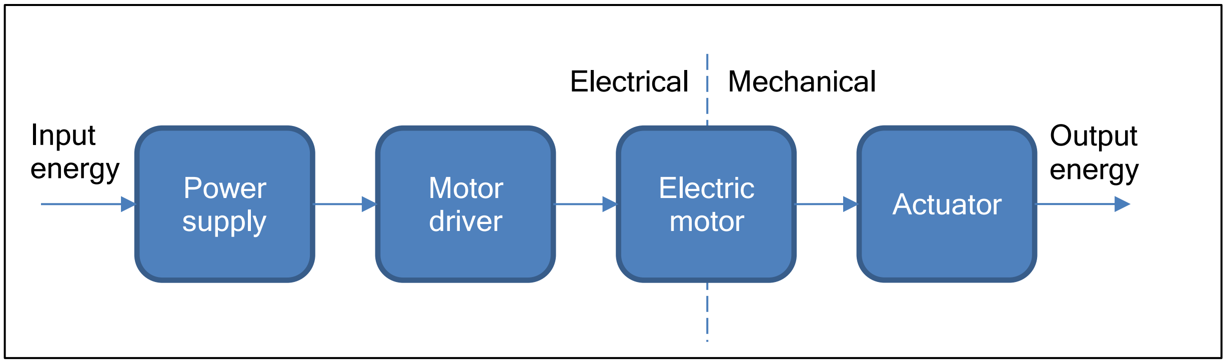

Figure 2
Electrical to mechanical energy conversion

A moving object possesses kinetic energy. When a motor decelerates a moving object, the energy returned to the system has to go somewhere. Similarly, potential energy in the form of gravitational forces, springs, etc., can be returned to the system as objects move. The energy is passed to the motor, which converts the mechanical energy back to electrical energy. The motor driver converts the electrical energy from the motor and returns it to the power bus between it and the power supply. At this point, something must be done with the remaining energy (fig. 3).

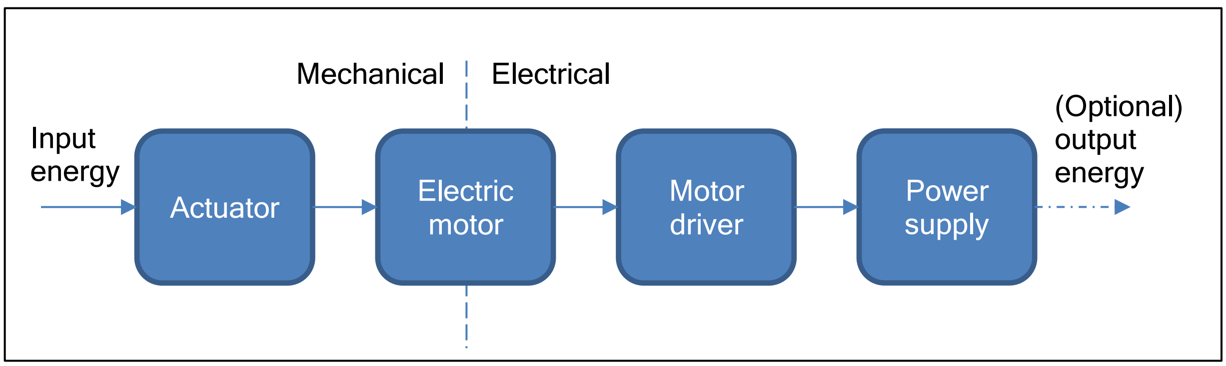

Figure 3
Mechanical to electrical energy conversion

Similar to the motoring scenario, the conversion process is not 100% efficient, and a portion of the regenerated energy is lost in the system as heat. There are several methods that can be used to handle the remaining energy. In some cases, it can be returned back to the power source (batteries, grid, etc.). If the energy is not removed from the system, the supply voltage will rise as the energy charges the bus capacitance. If the voltage rises too high, it could exceed voltage ratings of components and cause damage. To address this issue, there are several options that will be discussed.

METHODS, ASSUMPTIONS, AND PROCEDURES

Conventions and Variable Definitions

Before describing the formulas to solve the aforementioned problems, it is important to define the conventions and variables to be used. Table 1 defines the variables and units used in the subsequent calculations. All calculations are performed using the International System of Units (SI).

Table 1
Variable definitions

Variable	Definition
E	Energy (joules)
J	Moment of inertia (kilogram-meters^2)
V	Voltage (volts)
i	Current (amperes)
ω	Angular velocity (radians/second)
α	Angular acceleration (radians/second^2)
T	Torque (Newton-meters)
K_t	Torque constant of motor (Newton-meters/ampere)
K_e	Voltage constant of motor (volts/ω)
R	Resistance (ohms)
C	Capacitance (farads)
t	Time (seconds)
P	Power (watts)
Q	Electric charge (coulombs)
η	Efficiency (ratio less than one)
H	Corrected efficiency (represents η or $1/\eta$ depending on state)
r	Speed ratio between two points

Assumptions

The equations and methods that follow all operate under the assumption that acceleration is constant and current flow is modeled in a steady state. A simple permanent magnet direct current motor is assumed. While the basic theory is the same for all motors, there is no special consideration for alternating current, variable field motors, and linear motors, etc.

All equations presented deal with rotational motion since that is the eventual motion of a rotary motor. Conversions from other types of motion systems are left to the reader. Torque calculations are provided for inertial loads only. The calculations necessary to account for additional torque required for loads, gravity, springs, etc., are not presented and left to be factored in by the reader.

Since power can flow in both directions, a convention must be established for positive and negative power flow. This report assumes that power flowing from the electrical system to the load while motoring is positive and regenerative power is negative. When decelerating a load, a negative acceleration value shall be used resulting in a negative power value. For the purpose of power calculations, angular velocity shall be represented with a positive value without consideration of direction.

Fundamental Equations

The torque required to accelerate (or decelerate) a load is a function of the inertia of the load, and the acceleration rate, and can be found using equation 1. The moment of inertia must be a composite value of all rotating masses including the load, actuator components, and the motor rotor. Both the moment of inertia and the acceleration must be reflected to the same point (i.e., at the load) in order to determine the torque at that point in the mechanical system.

$$T_I = J\alpha \tag{1}$$

In addition to the torque required to accelerate, there is a load torque, T_L. The load torque is a composite value representing the torque required to overcome friction, gravity, and/or any other external force requiring torque to overcome. The total torque required is the sum of the inertial torque and load torque.

$$T = T_I + T_L \tag{2}$$

The rate that energy (or work) is performed is measured in joules per second, or watts. In a rotating object, power is the product of the torque acting on the object and its angular velocity.

$$P_{MECH} = T\omega \tag{3}$$

It is important to note that the mechanical process that converts motion between the motor and the final output is not perfectly efficient. Some portion of the mechanical power will be converted to heat, rather than motion, due to efficiency losses. While efficiency can vary with torque, temperature, speed, and other factors, these values can typically be incorporated into a conservative composite efficiency value that provides reasonably accurate results.

Since this report treats power as a vector, with positive and negative values, efficiency values are applied differently when motoring versus braking. To accommodate this convention, the symbol H is used to denote the correct form of η. When motoring, the inverse of the efficiency is applied resulting in a higher input power required to get the desired output. When regenerating, the efficiency is applied directly, resulting in less net energy being returned to the power supply.

$$if\ P > 0\ (Motoring):\ H = \frac{1}{\eta} \quad else\ (Braking): H = \eta \qquad P_{\mathrm{ADJUSTED}} = HP \tag{4}$$

The motor current associated with a given torque is a function of the torque at the motor and the motor torque constant as shown in equation 5. Note that the torque value must be reflected to the motor rotor with any efficiency losses accounted for.

$$i = \frac{T}{K_t} \tag{5}$$

In electrical terms, power is the product of voltage and current. In systems involving a resistance, the power equation can be further expanded as shown

$$P_{ELEC} = Vi = i^2R = \frac{V^2}{R} \tag{6}$$

The total voltage across the motor is the sum of the back EMF and the voltage across the resistance of the motor's windings. The back EMF is obtained by multiplying the motor's angular velocity by its voltage constant K_e. The voltage across the windings is the product of the current and the winding resistance.

$$V_{MOTOR} = K_e\omega + iR_M \tag{7}$$

The electrical power of the motor is the product of the motor voltage and the current traveling through its windings. The predominant efficiency loss on the electrical side of the motor is caused by the power lost as heat in the resistance of the windings. When motoring, the efficiency losses result in a greater amount of electrical power being required for a given mechanical output. When

regenerating, the loss results in less power returned to the power bus. The electrical power at the motor is calculated using equations 8 and 9. The first form of the equation solves power for a given angular velocity. The second form solves power for a given time when starting at an initial angular velocity and accelerating or decelerating at a constant rate.

$$P_{MOTOR\_ELEC}(\omega) = H(iK_e\omega + i^2R_M) \tag{8}$$

$$P_{MOTOR\_ELEC}(t) = H(iK_e(\omega_{t0} + \alpha t) + i^2R_M) \tag{9}$$

By integrating equation 9 with respect to time, the energy accumulated over a given time can be calculated as shown in equation 10. Initial stored energy is represented by E_{t0}. Since the convention used in this report is positive power flowing out of the electrical system, energy stored in the electrical system must be subtracted from as power leaves the electrical system and is converted to motion. Conversely, regenerative energy will be added to the initial energy. Note that if the current (torque) value changes, each period of time at a fixed current must be integrated separately. This is also true at any points where power transitions from motoring to regenerating due to the H value changing. At transition points, the time would be reset to zero, and the present values at that time would be entered for E_{t0} and ω_{t0}.

$$\int_0^t P(t)dt = E(t) = E_{t0} - H\left(\frac{1}{2}iK_e\alpha t^2 + (iK_e\omega_{t0} + i^2R_M)t\right) \tag{10}$$

This report focuses on the case where energy is stored electrically using capacitors. Energy stored by a capacitor is calculated using equation 11. The initial stored energy is determined by entering the capacitance of the storage system and the bus voltage.

$$E_{CAP} = \frac{1}{2}CV^2 \tag{11}$$

When decelerating, there is a certain velocity at which the power returned to the bus will be zero. This is due to the fact that regenerative power generated by the motor ramps down as the velocity approaches zero, while the power lost as heat in the motor stays constant with constant current (torque). By setting the power value in equation 9 to zero, and solving for t, the following solution is achieved. For the purpose of regeneration, solving this equation provides the point in time when the peak regenerative energy is reached.

$$t_{POW\_0} = -\frac{\omega_{t0}}{\alpha} - \frac{iR_M}{K_e\alpha} \tag{12}$$

This is further illustrated in figure 4, which graphs angular velocity and power versus time. Regenerative power is shown to be highest when deceleration first starts from the initial velocity. Power decreases linearly, reaching zero after a period of t_{POW_0} seconds. The red shaded area under the power curve represents the net regenerated energy.

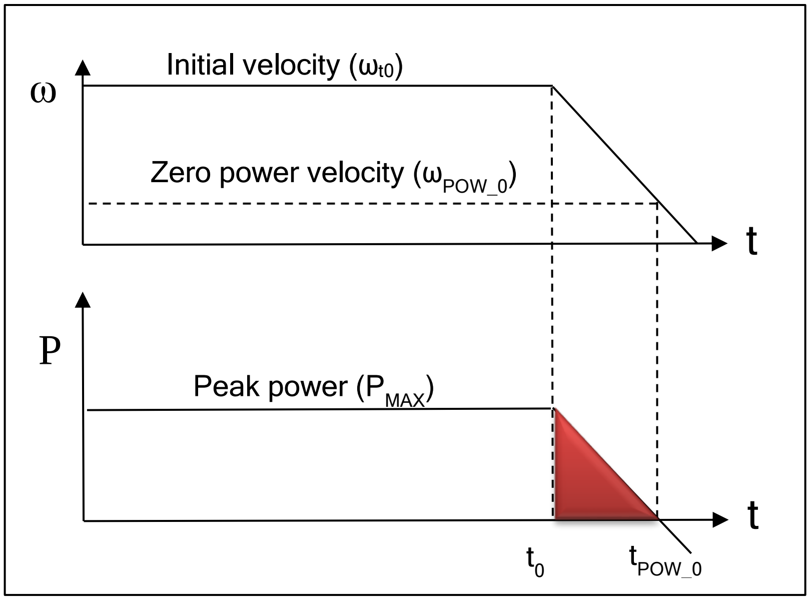

Figure 4
Power generation

The Euler Method

The previous equations did not account for copper losses on the supply side caused by capacitor equivalent series resistance (ESR) and resistance in the wires between the driver and capacitor. If these values are negligible, the equations previously described are sufficient. If these values are significant, the additional calculations provided in this section can be used.

A four quadrant motor driver behaves like a bidirectional DC/DC converter. Power on the motor side will be mirrored (minus efficiency losses) on the supply side, but at a different voltage, and hence different current (since power is the product of voltage and current). The current on the supply side is equal to the power (adjusted for driver efficiency losses) divided by the supply bus voltage. This would be a simple calculation if not for the fact that the supply voltage changes as energy is added and removed. Equation 8 provided a solution for the motor electrical power. On the bus, that power is equated to the sum of the resistive losses (i^2R) and the power being stored in the bus capacitance. This can be calculated using equation 13 which represents capacitor power and current in terms of electric charge. The derivative of Q with respect to time, represented as Q', is another way of representing electric current and Q/C equates to the voltage on a capacitor. The power is adjusted for efficiency losses (H_D) in the drive. Note that equation 13 effectively reverses the power convention so regenerative power is treated as a positive value as it adds energy, while motoring power is negated as energy is removed.

$$R_C Q'^2 + \frac{Q}{C} Q' = -H_D P_{MOTOR\_ELEC}(t) \tag{13}$$

Equation 13 is a nonlinear differential equation for which a symbolic solution is not known. However, since the initial value of Q can be solved for, an approximate numerical solution is possible using the Euler method. The equation to solve for Q' is represented in equation 14 using the quadratic equation.

$$Q' = F(t,Q) = \frac{-\frac{Q}{C} \pm \sqrt{\left(\frac{Q}{C}\right)^2 + 4R_C H_D P_{MOTOR\_ELEC}(t)}}{2R_C} \tag{14}$$

The initial value for Q is calculated using equation 15 with V set to the initial power supply bus voltage.

$$Q = VC \tag{15}$$

A time step size of value h must be selected as the interval at which to calculate each value. The smaller the step size (higher sampling frequency), the closer the approximation will be to the true solution. The general form of the equation is shown as

$$Q(n+1) = Q(n) + hF(t,Q) + hi_{CHRG}(n) + hi_{SHUNT}(n) \qquad where\ \ t_n = nh \tag{16}$$

Included in the equation is a function i_{CHRG}. This value represents the current supplied by the power supply to keep the capacitor charged. This would typically be supplied by a current limited voltage regulated power supply. During regeneration, the supply will stop supplying current when the voltage exceeds the supply's set value. When motoring, energy will be drawn from the capacitor, thus lowering the voltage, at which point the power supply will supply current to maintain its voltage. Ideally, the power supply would supply a current equal in magnitude to the consumed current Q'. However, resistance between the supply and capacitor will result in a voltage drop, causing a lesser value than Q'. This can be reduced by minimizing the distance between supply and capacitor, using thicker conductors to reduce resistance or running non-current carrying sense lines from the supply to the capacitor. Note that in the case of using sense lines, the supply should be capable of sourcing a voltage equal to the setpoint voltage, plus the product of the supply's current setpoint, and the resistance of the current carrying conductors.

$$i_{CHRG}(n) = \max\left(\min\left(\frac{V_{SUPPLY} - {}^{Q(n)}/_{C}}{R_{SUPPLY}}, i_{SUPPLY}\right), 0\right) \tag{17}$$

Also included in equation 16 is a function called i_{SHUNT}. This value represents the current pulled from the system when the dissipative shunt is applied. A simple on/off equation is represented in equation 17. When the voltage on the drive exceeds the upper limit, the shunt is applied across the power bus and consumes current, converting the electrical energy to heat. The equation can be modified to include hysteresis.

$$if\ v(n) > V_{UL}:\ i_{SHUNT}(n) = -\frac{V_{DRIVE}}{R_{SHUNT}} \qquad else:\ \ i_{SHUNT}(n) = 0 \tag{18}$$

Using the initial Q value, the solution for Q' at t = 0 can be calculated. Equation 16 is then used to solve Q at the next interval of time. This process is repeated until the desired t value has been reached. Equation 15 can be used to solve for the bus voltage at any point in time.

Power Supply Requirements

There are several considerations when designing/selecting a power supply for an electrically actuated system. The most basic parameters to consider are voltage and current. For a given motor, a minimum voltage value is required to obtain a given maximum angular velocity, and a minimum current value is required to obtain a given maximum torque.

Solving equation 7 using maximum angular velocity and current (based on maximum motor torque) will determine the minimum voltage required to achieve that velocity and torque. This process is repeated for all motors powered by the supply, with the highest voltage being the minimum supply bus voltage required. Note that the resistance in the wiring between the driver and motor should be included in the winding resistance when performing the calculation. The calculated value is an absolute minimum. Additional margin should be factored in to account for efficiency losses, voltage drops, tolerances, etc.

Minimum current requirements can be determined by first determining the worst case operating scenario for the system with regard to power consumption. Electrical power can be calculated using equation 8 for the various scenarios, summing power consumptions from multiple motors if necessary. Equation 6 can then be used to calculate current on the power supply factoring in efficiency of the drivers. Again, additional headroom should be factored into the final current rating.

$$i_{SUPPLY} = \frac{P_{MOTOR\_ELEC\_MAX}}{V_{BUS}\eta_{DRVR}} \tag{19}$$

While the voltage requirements are fixed, a high power stored energy system can be used to reduce the peak current loads on the power supply by storing energy at a lower power and delivering high power to the system momentarily as needed. One means of accomplishing this is by adding additional capacitance to the power bus. This has the added benefit that the capacitance can also store regenerative energy. If power is consumed by the actuators at a rate higher than the power supply is providing, the bus voltage will drop as the net energy in the capacitors is reduced. This voltage drop must be taken into account when determining the nominal bus voltage to ensure the minimum voltage required to operate the system is never reached. Another important consideration when adding capacitance is the fact that a discharged capacitor, when connected to a voltage source, appears as a short. The power supply must be capable of charging the capacitance to the nominal bus voltage in a controlled manner (i.e., constant current or constant power). The supply must also be tolerant of bus voltages exceeding its set value due to regenerative charging.

Approaches to Handling Regenerative Energy

There are several approaches to managing regenerative energy. One approach is to reduce the amount of energy produced. By reducing the maximum velocity, the total kinetic energy of the system is reduced, resulting in less net energy returned to the system when decelerating. Reducing the deceleration rate while maintaining the same initial velocity will actually result in a net increase in energy returned to the system due to reduced resistive losses. However, that energy will be returned to the system over a greater period of time and at a lower maximum power. If there are predictable loads on the power bus that could absorb this lower power energy, this could also be a suitable approach.

Another approach is to manage the voltage increase that results from energy being added to the power bus. Using a given capacitance, one could lower the bus voltage, thus requiring more energy to get to the same peak voltage. This approach requires that the voltage does not drop below the minimum required voltage calculated earlier. Another option that maintains a given bus capacitance is to use components with a higher voltage rating, thus allowing the regenerated energy to increase bus voltage to a higher value without damage. However, the voltage ratings required to achieve this may result in components that are expensive, difficult to source, and/or much larger in size.

Storing the energy temporarily is another option. There are numerous methods of storing energy (batteries, flywheels, etc.). This report will focus on the use of additional capacitance as a

simple means of energy storage. If the capacitance on the bus is increased, the same amount of energy will result in a smaller increase in voltage. However, the relationship between capacitance and voltage is nonlinear, meaning the reduction in voltage rise will diminish, as additional capacitance is added.

In some cases, it may be possible to return the regenerative energy to the power source. This could be another power bus (AC or DC), batteries, etc. This approach is similar to the energy storage approach mentioned previously but passes the burden of handling the excess energy to an external system. The challenge in this approach (assuming the power source permits it) is that the power supply is made more complicated by being required to operate bi directionally.

The final option discussed in this report is to remove excess energy from the system through a dissipative load. The basic idea of a shunt regulator is shown in figure 5. A switch is turned on when the supply voltage exceeds a predetermined threshold. This puts a resistive load in parallel with the supply, and causes the excess energy to dissipate through the load, converting that energy to heat. Once the voltage drops to a given level, the switch is opened, disconnecting the load. This is a simple and effective technique with its main drawback being that the energy is wasted as heat rather than being recovered for future use. It also adds additional thermal requirements as that heat eventually needs to be removed from the system.

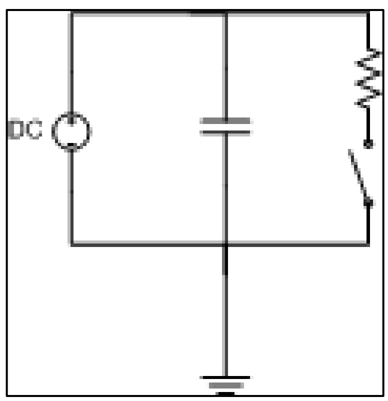

Figure 5
Shunt regulator

There are two values that need to be calculated when choosing a load resistor. First, the load resistance needs to be chosen so it has a higher power dissipation than the peak power. A conservative peak power value can be calculated using equation 8 at the maximum angular velocity.

The resistance value needed to dissipate a given power at a given peak voltage is expressed using equation 6. The chosen resistance value must be equal to or less than this value.

$$R_{LOAD} <= \frac{V_{PEAK}^2}{|P|} \tag{20}$$

If the bus capacitance is able to absorb a significant portion of the regenerative energy before hitting the peak allowable voltage, the energy it absorbs no longer needs to be dissipated. The load resistor would only be required to dissipate energy in excess of the energy that the capacitor is able to recapture. Using equation 11, one can calculate the amount of energy that brings the capacitance from the nominal voltage to the peak bus voltage. Plugging that value into equation 10 and solving for t using the quadratic equation gives the time at which the capacitor voltage will reach its peak.

$$t = \frac{P_{ELEC}(\omega_{t0}) \pm \sqrt{(-P_{ELEC}(\omega_{t0}))^2 + 2HiK_e\alpha E_{CAP}}}{HiK_e\alpha} \tag{21}$$

Solving equation 9, using the calculated time value, gives the regenerative power at that point in time. Equation 19 can then be solved using the new power value resulting in a higher minimum resistance value and hence a lower power output.

The other load value that needs to be calculated is the resistor's power rating. A conservative solution would be to use the power rating calculated previously. However, resistors often provide both a continuous power rating and an intermittent rating for higher power dissipation occurring during a finite period. Since the regenerative power starts at a peak value and quickly drops, it is possible to size the load by averaging the power over the period of regeneration. If periods of regenerative deceleration are infrequent, the power rating could potentially be reduced further.

Conductor Selection

The conductors that connect the power supply, drivers, and motors must be capable of moving the required current and be rated to the anticipated voltages. Any resistance in the conductors will be a source of power loss (i^2R) and result in a drop in voltage (iR). Power lost in the conductors is converted to heat. As conductors heat, their resistance increases, further compounding the problem. It is essential that resistance in conductors is considered in the design. Resistance is a function of the material conductivity, cross-sectional area, length, and temperature. The most common conductors are copper wires, but bus bars and printed circuit boards are common as well. Measurement of an existing system is the simplest and most accurate method of obtaining resistance, but calculations are also relatively simple using available equations and lookup tables. Being that motors and the power required to control them are very electrically noisy, it is also important to shield motor conductors and attempt to isolate them from any other signals that may be susceptible to noise.

RESULTS AND DISCUSSIONS

In order to demonstrate the use of these equations in practice, an example is presented in this section.

Example of how Equations are Applied

As shown in table 2, a single axis rotary actuation system has been designed and the evaluation of its electrical requirements must be performed. The maximum velocity and acceleration values have been specified and cannot be changed. Actuator efficiency, moment of intertia, gear ratio, and load torque (constant in this example) at the motor have been provided by the actuator designer. Motor moment of inertia, winding resistance, and torque constants have been pulled from the motor specification sheet, and resistance and capacitance of the power bus have been measured. The efficiency of the motor driver has been pulled from its specification sheet.

Table 2
Example data

Data	Variable	Value	Source
Peak output speed	ω_{OUT}	18 deg/s	Design decision (at actuator output)
Output deceleration	α_{OUT}	25 deg/s^2	Design decision (at actuator output)
Actuator moment of inertia	J_{ACT}	0.012kg-m^2	Calculated by actuator designer
Actuator efficiency	η_{ACT}	0.87	Calculated by actuator designer
Actuator gear ratio	r	840:1	Calculated by actuator designer
Load torque	T_L	200 Nm	Calculated by actuator designer
Motor torque constant	K_t	0.15 Nm/A	Motor specification
Motor winding resistance	R_{MOTOR}	0.1 ohms	Motor specification
Motor moment of inertia	J_{MOTOR}	0.0003 kg-m^2	Motor specification
Supply capacitance	C	0.005 F	Measured on power bus
Capacitor to driver resistance	R_{BUS}	0.004 ohms	Measured on power bus
Supply to capacitor resistance	R_{SUPPLY}	0.1 ohms	Measured on power bus
Driver efficiency	η_{DRVR}	0.98	From driver datasheet.

The first step is to determine the power supply voltage requirement. Start by converting speed and acceleration values to the correct units and reflect the values back to the motor through the gear ratio.

$$\omega_{MAX} = \omega_{OUT} r \frac{\pi}{180} = 263.9\, rads/s \tag{22}$$

$$\alpha = \alpha_{OUT} r \frac{\pi}{180} = 366.5\, rads/s^2 \tag{23}$$

Next, calculate the torque and peak mechanical power at the motor both when motoring and when regenerating. This is accomplished using equations 1 through 4.

$$Motoring\colon\ P_{MOTOR\_MECH} = \frac{T\omega}{\eta_{ACT}} = \frac{\left(\left((.012 + .0003)(366.5)\right) + \frac{200}{840}\right)(263.9)}{.87} = 1439.6W \tag{24}$$

$$Regen\colon\ P_{MOTOR\_MECH} = \eta_{ACT} T\omega = (.87)\left(\left((.0123)(-366.5)\right) + \frac{200}{840}\right)(263.9) = -980.3\text{W} \tag{25}$$

A torque value is calculated for the motoring power and then solved for current using equation 5. This is the current required from the driver to deliver the torque required to overcome the load torque and accelerate the moment of inertia at the given rate.

$$i_{MOTORING} = \frac{T}{K_t} = \frac{P_{MOTOR\_MECH}/\omega_{MAX}}{K_t} = \frac{1439.6/263.9}{.15} = 36.37A \tag{26}$$

The maximum angular velocity and current can now be entered into equation 7 to determine the minimum required motor voltage to be supplied by the drive.

$$V_{MOTOR} = K_e\omega + iR_M = (.15)(263.9) + (36.37)(.1 + .004) = 43.37V \tag{27}$$

Adding an additional 10% of headroom and rounding up gives a nominal bus voltage of 48 V. Equations 8 and 18 can then be used to determine maximum electrical power and bus current.

$$P_{ELEC\_MAX} = \frac{iK_e\omega + i^2R_M}{\eta_{DRVR}} = \frac{(36.37)(.15)(263.9) + (36.37^2)(.1)}{.98} = 1604.1W \tag{28}$$

$$i_{SUPPLY} = \frac{P_{ELEC\_MAX}}{V_{MOTOR}} = \frac{1604.1}{43.37} = 37A \tag{29}$$

Adding 10% results in a current requirement for the supply of 40.7 A, which is rounded up to 45 A. At this point, consideration can be given to the regenerative energy while decelerating. The regenerative mechanical power of the motor calculated in equation 25 is first used to obtain a motor current. This is the current flowing from the motor to the driver when the motor is decelerated at the given rate.

$$i_{REGEN} = \frac{T}{K_t} = \frac{P_{MOTOR\_MECH}/\omega_{MAX}}{K_t} = \frac{-980.3/263.9}{.15} = -24.76A \tag{30}$$

Equation 8 is used to find the peak regenerative electrical power.

$$P_{REGEN\_PEAK} = \eta_{DRVR}(iK_e\omega + i^2R_M) = .98\big((-24.76)(39.59) + (-24.76^2)(.1)\big) = -900.4W \tag{31}$$

Equation 12 is then used to find the time when regeneration completes, and equation 11 is used to find the energy stored in the bus capacitance at the nominal bus voltage. The time is fed into equation 10 to obtain the peak energy.

$$t_{POW\_0} = -\frac{\omega_{t0}}{\alpha} - \frac{iR_M}{K_e\alpha} = -\frac{263.9}{-366.5} - \frac{(-24.76)(.1)}{(.15)(-366.5)} = .675\ s \tag{32}$$

$$E_{t0} = \frac{1}{2}CV^2 = \frac{1}{2}(.005)(48^2) = 5.76J \tag{33}$$

$$E_{MAX} = 5.76 - .98\Bigg(\frac{1}{2}(-24.76)(.15)(-366.5)(.675^2) + \big((-24.76)(.15)(263.9) + (-24.76^2)(.1)\big)(.675)\Bigg) = 309.7\ J \tag{34}$$

With 5 millifarads of capacitance at the nominal bus voltage, the stored energy of 5.76 J is increased by 303.9 to 309.7 J when decelerated to a stop from the peak angular velocity. Equation 11 can be used to determine the bus voltage at the calculated energy.

$$V_{PEAK} = \sqrt{\frac{2E}{C}} = \sqrt{\frac{(2)(309.7)}{.005}} = 352\ V \tag{35}$$

This voltage greatly exceeds the nominal bus voltage. The example assumes the additional space and cost of implementing components tolerant of a voltage that high is unacceptable. Additional remediation is required. The first step is to look at adding capacitance. Graphing the voltage rise to capacitance provides the graph shown in figure 6.

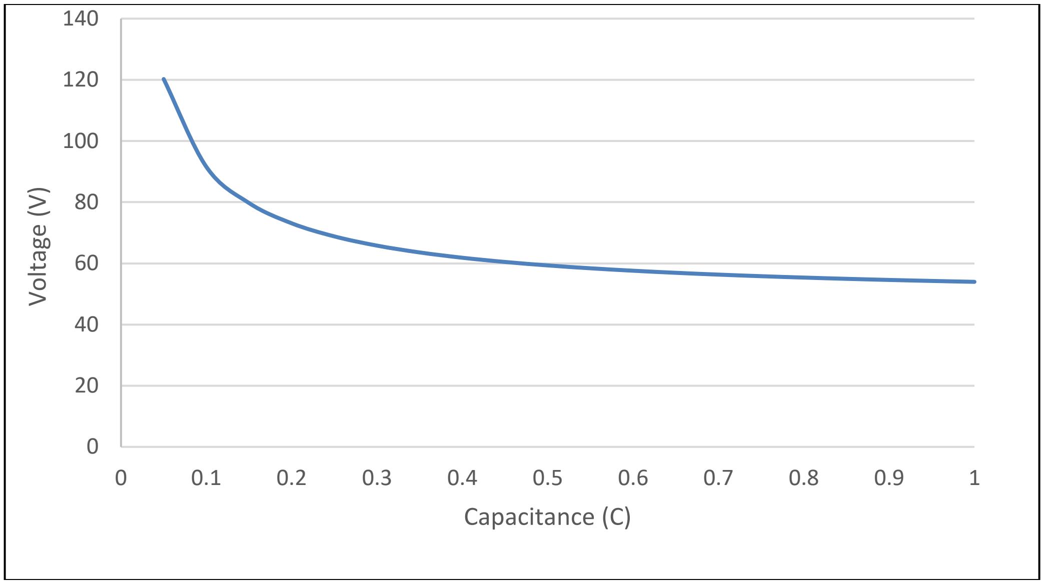

Figure 6
Bus voltage versus capacitance (E = 303.9 J, VSUPPLY = 48 V)

Observing the graph, 60 V is chosen as a reasonable balance between voltage rise and additional capacitance. Equation 11 is used to solve for capacitance C with the given energy increase. The higher capacitance will result in more stored energy at the nominal bus voltage which is accounted for in equation 35.

$$E = \frac{1}{2}CV_{PEAK}^2 - \frac{1}{2}CV_{SUPPLY}^2 \Rightarrow C = \frac{2E}{V_{PEAK}^2 - V_{SUPPLY}^2} = \frac{2(303.9)}{(60)^2 - (48)^2} = 0.47F \quad (36)$$

This example assumes that there is insufficient space to fit a half farad of capacitance. However, the system does have space available for a quarter farad of capacitance at 60 V. Using equation 11, the energy can be calculated stored in 0.25 F capacitance at 60 V, and subtract from that the energy at the bus voltage of 48 V to determine how much excess energy can be absorbed before achieving the peak allowable voltage.

$$E = \frac{1}{2}C(V_{PEAK}^2 - V_{SUPPLY}^2) = \frac{1}{2}(.25)(60^2 - 48^2) = 162\,Joules \quad (37)$$

Of the 303.9 J of energy generated, the additional capacitance can handle 162 J, leaving 141.9 J that still needs to be handled. This excess energy will be dissipated by a load resistor. Equation 20 is used to solve for the time to deliver 162 J of energy to the power bus.

$$t = \frac{-900.4 \pm \sqrt{900.4^2 + (2)(.98)(-24.76)(.15)(-366.5)(162)}}{(.98)(-24.76)(.15)(-366.5)} = 0.16, \cancel{-1.5} \quad (38)$$

The correct time is 0.16 sec. This is then entered into equation 9, and the power at that time is calculated.

$$P(.16) = (.98)\left((-24.76)(.15)\big(263.9 + (-366.5)(.16)\big) + (-24.76)^2(.1)\right) = -728.9W \quad (39)$$

With the load power calculated, equation 19 is used to determine the maximum load resistance required.

$$R_{LOAD} <= \frac{{V_{PEAK}}^2}{|P|} = \frac{60^2}{728.9} = 4.94\ Ohms \tag{40}$$

In addition, circuitry is needed that will apply the bus voltage to the load when the bus voltage reaches 60 V and disconnect the load when it drops below an acceptable value. Some degree of hysteresis should be implemented to reduce the switching losses. However, too low of a cutoff value will result in excess power dissipated as heat that could instead be stored and consumed for future motor driving. Furthermore, the component responsible for switching must be rated to handle the current that will pass through it and the load. While the component itself will drop some amount of voltage (as well as dissipate some of the energy), the current can be safely approximated using Ohm's law with the peak voltage and load resistance.

$$i = \frac{V}{R} = \frac{60}{4.94} = 12.15\ Amperes \tag{41}$$

Now solutions will be presented using the Euler method to also account for the power supply bus resistance and capacitor ESR. Assuming that the 0.25 farad capacitor has an ESR of 22 milliohms, combined with the bus resistance that equals a total of 26 milliohms. Equation 15 is used to solve for Q_0. Equations 14 and 16 are then used to numerically solve for Q and Q' using a step size h of 0.01 sec.

$$Q_0 = V_{SUPPLY}C = (48)(.25) = 12C \tag{42}$$

Table 3 shows a worst case acceleration from zero to the maximum velocity. In this case, the energy in the capacitor is being removed in order to provide power to the motor, while the power supply attempts to replenish the lost energy using current (i_{CHRG}) from equation 17. As one would expect, there is little charge current applied initially as the lower speed results in a small power consumption. However, as the motor accelerates to its peak speed, the power draw increases significantly, requiring more current from the power supply to maintain the bus voltage. It's also apparent from this chart how significant the power loss in the wiring can be. At 37 A, there is a 35.6 W power loss due to resistance in the cabling and capacitor as well as nearly a 1 V drop in voltage.

Table 3
Example motoring Euler method

Time	Speed	Bus power	Charge current	Q	Q'	Capacitor voltage	Drive voltage	Capacitor power	Resistance power
0.00	0.00	134.96	0.00	12.00	-2.82	48.00	47.93	-135.17	0.21
0.01	3.67	155.37	0.94	11.97	-3.25	47.89	47.80	-155.64	0.27
0.02	7.33	175.77	1.71	11.95	-3.68	47.79	47.70	-176.12	0.35
0.03	11.00	196.17	2.37	11.93	-4.12	47.72	47.61	-196.61	0.44
0.04	14.66	216.57	2.95	11.91	-4.56	47.65	47.53	-217.11	0.54
0.05	18.33	236.97	3.49	11.90	-4.99	47.58	47.45	-237.62	0.65
0.06	21.99	257.37	3.99	11.88	-5.43	47.52	47.38	-258.14	0.77
0.07	25.66	277.77	4.47	11.87	-5.87	47.46	47.31	-278.67	0.90
0.08	29.32	298.18	4.94	11.85	-6.31	47.41	47.24	-299.21	1.04
0.09	32.99	318.58	5.40	11.84	-6.75	47.35	47.18	-319.76	1.19
0.10	36.65	338.98	5.85	11.82	-7.20	47.30	47.11	-340.32	1.35
0.11	40.32	359.38	6.30	11.81	-7.64	47.24	47.05	-360.90	1.52
0.12	43.98	379.78	6.74	11.80	-8.08	47.19	46.98	-381.48	1.70
0.13	47.65	400.18	7.19	11.78	-8.53	47.14	46.92	-402.07	1.89
0.14	51.31	420.58	7.64	11.77	-8.98	47.08	46.85	-422.68	2.10
0.15	54.98	440.98	8.08	11.76	-9.43	47.03	46.78	-443.29	2.31
…	…	…		…	…	…	…	…	…
0.67	245.56	1501.86	33.25	11.00	-34.84	44.01	43.10	-1533.42	31.56
0.68	249.22	1522.26	33.78	10.99	-35.38	43.95	43.03	-1554.81	32.54
0.69	252.89	1542.66	34.31	10.97	-35.92	43.88	42.95	-1576.21	33.54
0.70	256.55	1563.06	34.85	10.95	-36.46	43.82	42.87	-1597.63	34.56
0.71	260.22	1583.46	35.39	10.94	-37.00	43.75	42.79	-1619.07	35.60
0.72	263.88	1603.87	35.93	10.92	-37.55	43.69	42.71	-1640.53	36.66

Of special note is that the capacitor voltage is slowly decreasing despite the fact that the power supply never reaches its current limit. This is due to the resistance between the supply and capacitor. As the current output increases, the voltage drop across that resistance also increases. The end effect is that the actual current available for a given supply voltage and line resistance is a function of the difference in voltage between the supply and the capacitor. In this example, the less than expected charge current, as well as the voltage, drops in the lines to the drive, results in a worst case voltage at the drive of 42.7 V. This is below the minimum voltage, calculated in equation 26, necessary to attain peak speed. The solution is to reduce the supply to capacitor conductor resistance or change to a higher bus voltage. By increasing the bus voltage to 50 V, the worst case voltage seen at the drive is 45 V. Also of note, is the fact that the lower charge currents allow a reduction of the current rating of the power supply. At 50 V, the worst case charge current is 34 A. Using the spreadsheet to adjust supply current values, it was found that the maximum charge current could be reduced to 28 A before hitting the minimum voltage limit.

With worst case motoring calculated, it's time to look at regeneration. Using the new bus voltage of 50 V, table 4 shows a worst case deceleration from maximum velocity to zero. The Q' values in this example are significantly reduced from the motoring example. The reason for this is that whereas when motoring, the inefficiencies in the system resulted in additional power being required from the supply, when regenerating, those same inefficiencies result in less regenerative

power being returned to the supply. In this particular example, the resistance of the bus has a negligible impact on the energy returned to the capacitor. However, with an initial current of 17.9 amperes (Q' is current), it's obvious that with even slightly higher resistance, values of a significant portion of the power can be lost prior to reaching the capacitor.

Table 4
Example regeneration Euler method without shunt

Time	Speed	Bus power	Shunt current	Q	Q'	Capacitor voltage	Drive voltage	Capacitor power	Resistance power
0.00	263.90	-919.00	0.00	12.50	17.85	50.00	50.46	892.34	8.28
0.01	260.24	-905.38	0.00	12.68	17.34	50.71	51.16	879.46	7.82
0.02	256.57	-891.77	0.00	12.85	16.86	51.41	51.85	866.55	7.39
0.03	252.91	-878.15	0.00	13.02	16.39	52.08	52.51	853.61	6.98
0.04	249.24	-864.54	0.00	13.18	15.94	52.74	53.15	840.64	6.61
0.05	245.58	-850.92	0.00	13.34	15.51	53.37	53.78	827.65	6.25
0.06	241.91	-837.31	0.00	13.50	15.09	54.00	54.39	814.65	5.92
0.07	238.25	-823.69	0.00	13.65	14.68	54.60	54.98	801.62	5.60
0.08	234.58	-810.08	0.00	13.80	14.29	55.19	55.56	788.57	5.31
0.09	230.92	-796.47	0.00	13.94	13.91	55.76	56.12	775.51	5.03
0.10	227.25	-782.85	0.00	14.08	13.54	56.31	56.67	762.43	4.77
0.11	223.59	-769.24	0.00	14.21	13.18	56.86	57.20	749.34	4.52
0.12	219.92	-755.62	0.00	14.35	12.83	57.38	57.72	736.23	4.28
0.13	216.26	-742.01	0.00	14.47	12.49	57.90	58.22	723.11	4.06
0.14	212.59	-728.39	0.00	14.60	12.16	58.40	58.71	709.98	3.84
0.15	208.93	-714.78	0.00	14.72	11.83	58.88	59.19	696.84	3.64
0.16	205.26	-687.14	0.00	14.84	11.52	59.36	59.65	683.69	3.45
0.17	201.60	-673.80	0.00	14.95	11.21	59.82	60.11	670.53	3.27
0.18	197.93	-660.46	0.00	15.07	10.91	60.26	60.55	657.36	3.09
0.19	194.27	-647.11	0.00	15.18	10.61	60.70	60.98	644.18	2.93
…	…	…	…	…	…	…	…	…	…
0.67	18.34	-6.68	0.00	17.62	0.09	70.46	70.46	6.68	0.00
0.68	14.68	6.94	0.00	17.62	-0.10	70.46	70.46	-6.94	0.00
0.69	11.01	20.83	0.00	17.62	-0.30	70.46	70.45	-20.83	0.00
0.70	7.35	34.72	0.00	17.61	-0.49	70.45	70.44	-34.73	0.01
0.71	3.68	48.61	0.00	17.61	-0.69	70.43	70.41	-48.63	0.01
0.72	0.02	62.51	0.00	17.60	-0.89	70.40	70.38	-62.53	0.02

At 0.18 s, the drive voltage exceeds 60 V, necessitating additional regeneration remediation. Figure 7 graphs the rise in voltage caused by the regenerative energy being added to the bus capacitance. By the time the motion has stopped, the bus voltage exceeds 70 V, well in excess of the 60 V limit. By recalculating equation 40, using the bus power at time 1.6 s, a resistance value of 5.2 ohms is obtained, which is then rounded down to 5 ohms. This 5 ohm load is applied across the power bus to regulate the bus voltage.

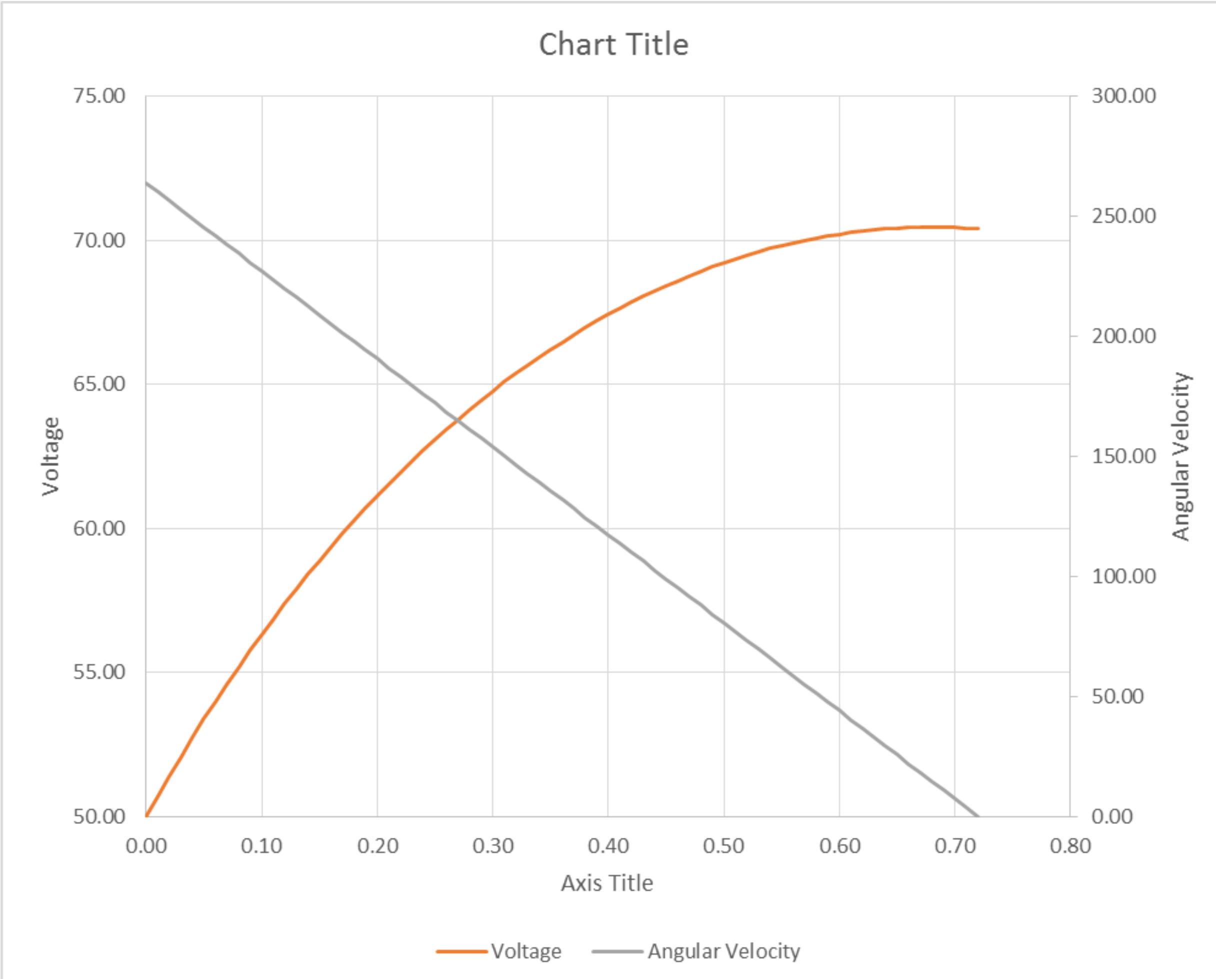

Figure 7
Regeneration without shunt regulation

By implementing a shunt regulator algorithm, the drive voltage can be prevented from exceeding the 60 V threshold. Figure 8 illustrates the same motion profile but with the 5-ohm shunt resistor switching on at 60 V and off at 59 V. Calculating the power and energy dissipated by the shunt is straightforward. For each time n, square i_{SHUNT} and multiply by the shunt resistance to find the power at that point in time. Multiply each power value by the h interval to determine the energy dissipated in that period. Finally, sum the energy values together to determine the total energy dissipated. For this example, the peak power seen in the shunt is 726 W and the total energy dissipated is 192 J. The average power over a 1.4 s acceleration/deceleration cycle would be 192/1.4 = 137 W. In this case, a 150 W power resistor would likely be adequate, though additional thermal factors would need to be evaluated as well.

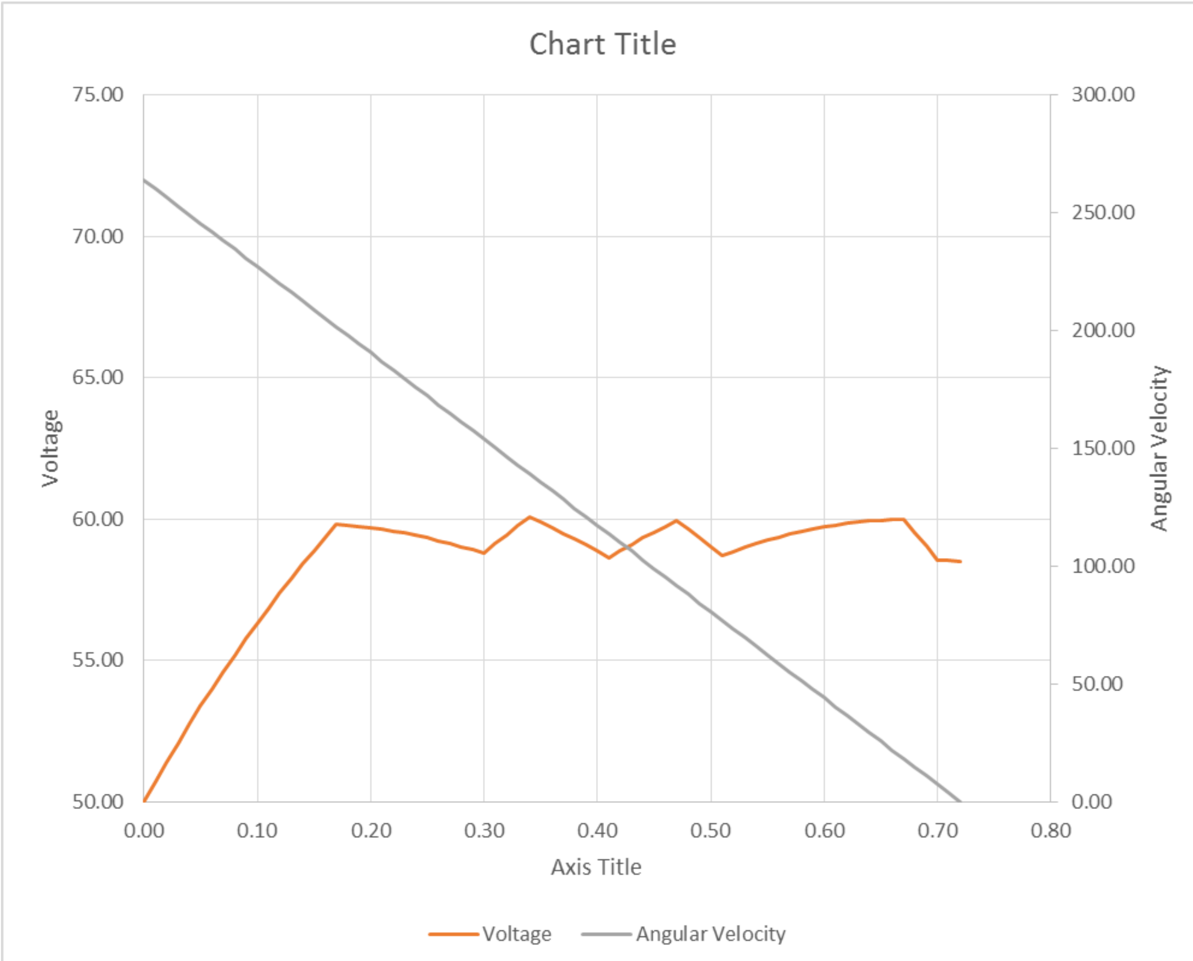

Figure 8
Regeneration with shunt regulator

CONCLUSIONS

This report has presented methods to develop electrical requirements for electro-mechanical actuation systems. Several topics were addressed including: power supply ratings, the effects of conductor resistance, and the issue of regenerative energy handling. All of the required equations have been listed and detailed. Several different approaches to addressing specific issues have been presented along with the pros and cons of each.

While the fundamental equations detailed previously are sufficient for estimations, the Euler method is more appropriate for complex systems, as it can take into account multiple actuators, varying torques, bus resistances, and numerous other factors. For the purpose of this report, an Excel spreadsheet was used. Other tools could be used as well (i.e., MATLAB, Visual Studio, and LabVIEW).

As with any engineering design, a factor of safety should be applied to calculated values. This would include components rated higher than the expected peak voltage, additional capacitance, load resistors that dissipate more power with a higher wattage than calculated, and a supply that has additional voltage and current headroom.

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